F36 Wavefront Analysis

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Abstract

The resolution of ground-based optical telescopes is limited by turbulence in the atmosphere, which causes distortion of incoming wavefronts. This phenomenon is called „seeing". To conquer this problem, adaptive optics has been invented, which uses systems that measure and correct these wavefront distortions in real time by comparing them to each other, thus improving image resolution.

In this experiment, the system gain of a CMOS camera system was measured, which was done by plotting the variance against the mean signal, also called the „photon transfer curve". The slope of this curve corresponds to the desired system gain in units of electrons per ADU. It was determined to be $g = 0,94$ e$^-$/ADU, which coincided with the provided value of $g_{script} = 0,944$ e$^-$/ADU.

In the second part, a Shack-Hartmann sensor was set up using a microlens array, a collimated laser source, and the CMOS detector from the first part. The positions of the centroids of the microlens array for the undistorted wavefront were recorded.

After adding a lens with an astigmatism defect to the system in order to distort the wavefront, the new centroids were recorded and compared to the previous „flat" ones. This allowed the Shack-Hartmann gradients to be computed, which, when plotted, appeared to have lost spherical symmetry, but still showed reflectional symmetry. Considering the nature of the astigmatism defect, this was to be expected.

The construction of the Shack-Hartmann gradients would be the first step in reconstructing the original wavefront. However, this lies beyond the scope of this report.

1. Introduction

1.1 Theoretical Background

When observing astral objects with an optical instrument, one can be confronted with a phenomenon called „seeing", which describes the contribution of fluctuations in the atmosphere to the distortion of incoming wavefronts. It can present itself as twinkling of stars or other variable distortions of the objects observed. Large-scale telescopes struggle with this, as there is a point at which the limitations set by the Rayleigh criterion become negligible and seeing becomes the main cause of a limited resolution. To work around this problem and enable more precise measurements, one can apply adaptive optics. Understanding the workings behind adaptive optics is crucial for today's astronomical observations, hence this experiment.

As mentioned, seeing is caused by turbulent air in the atmosphere. Temperature gradients create turbulence cells with different refractive indices, acting like weak, but numerous lenses. These „lenses" are randomly distributed in the atmosphere. Due to its constant fluctuations, the incoming wavefronts are refracted differently at various positions and times.

To correct this, adaptive optic systems have emerged. The idea behind them is to measure a wavefront, determine how distorted it is and then correcting it in real time. The correction is computed through comparison with previous measurements and can then be applied using deformable mirrors. This can improve the spatial resolution drastically, making sharp observations from earth's surface possible.

For the correction to be possible, the change of the wavefront (i. e. its gradients) has to be known. In this experiment, a Shack-Hartmann sensor will be used for this task.

The Shack-Hartmann sensor measures the wavefronts by dividing the aperture into smaller aspects using a microlens array. Each microlens has a focal point, the non-distorted position of which is known. The sensor then measures the displacement of an incoming wavefront from this reference point. The Shack-Hartmann gradients can then be computed, which together with Zernike polynomials can be used to reconstruct the undistorted wavefront.

The Shack-Hartmann Sensor used in this experiment uses a CMOS (Complementary Metal Oxide Semiconductor) camera to determine the positions of the focal points. Each pixel in this camera has its own amplifier, enabling faster readouts. These can be done individually by a connected ADC, creating an image (here in analog-digital-units, also called „ADU").

The CMOS camera has a photodiode and multiple transistors to integrate the incoming light.

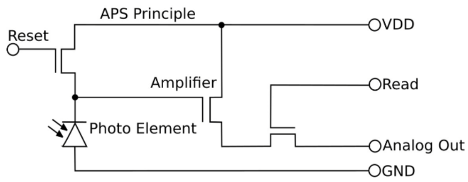

Figure 1.1: Schematic basic single active pixel in a CMOS sensor ([1])

Before exposure to light, the diode is charged using an external power source. Incoming photons discharge it via the photoelectric effect, making the voltage drop from the original level. The total voltage drop correlates with the incoming light's intensity. An analog-to-digital converter processes this analog voltage and returns a digital „count". Methods such as „pixel binning", where an arbitrary amount of pixels are interpreted as one, are used to improve the depth of the digital image. Since electrons can unintentionally move from the valence band to the conduction band due to thermal excitation, a „dark current bias" may have to be considered.

An important quantity for the final image is the the „system gain", which describes the ratio of the number of photoelectrons to the digital ADU count. Knowing the system gain allows for a conversion of ADU to analog photon counts. Lastly, astigmatism is a common refraction error caused by spherical asymmetry in a lens. It is usually corrected by lenses which are more cylindrical than spherical in nature [3].

1.2 Goals

First, the system gain of a CMOS detector will be determined, which is also referred to as measuring the photon transfer curve. For this, an adjustable flat-field lamp will be used.

In the second part, the Shack-Hartmann gradients of an astigmatism lens will be determined and interpreted.

2. Means of Measurement

In this part, the system gain of a CMOS camera is determined by plotting the mean signal S_C against the variance $N_C^2 - R_C^2$, where N_C is the total noise of the image and R_C is the readout noise of the chip, both in ADU. Since photon noise follows a Poisson distribution, the variance of the signal should be proportional to its mean. A line can be fitted through the corresponding data, allowing the system gain g to be determined by the slope of the fit.

This can be derived from the formula for the expected mean signal count:

$$S_C = \frac{-1 + \sqrt{1 + 4g^2k^2(N_C^2 - R_C^2)}}{2gk^2}$$

$$\Rightarrow \lim_{k\to 0} S_C = \lim_{k\to 0} \frac{-1 + \sqrt{1 + 4g^2k^2(N_C^2 - R_C^2)}}{2gk^2}$$

Using L'Hôpital's rule, it follows:

$$\begin{aligned} \Rightarrow \quad & \lim_{k\to 0} \frac{4g^2k}{4gk} \frac{(N_C^2 - R_C^2)}{\sqrt{1 + 4g^2k^2(N_C^2 - R_C^2)}} \\ &= g(N_C^2 - R_C^2) \end{aligned}$$

Thus, as the flat field variation k approaches 0, as would be the case in this experiment's setup, the plot should result in a straight line.

To remove pixel-dependent variations in sensitivity, which cause the so-called „flat field effect", two images were taken at the same intensity level and subtracted from one another. This cancels out the fixed pattern of the flat field effect, since both images have the same systematic pixel sensitivity regardless of statistical variation. To increase accuracy, the two images were taken in quick succession to ensure stable illumination between exposures.

Before the individual images can be subtracted from each other, they have to be normalized to have the same average intensity. After that, the dark current bias can also be subtracted from both images to get even more precise results, which is necessary if the bias is very large, thus non-negligible.

After canceling out the flat field effects, the standard deviation in a selected region can be computed and squared to get the variance. Since this is the variance of *two* subtracted images, it is also doubled, so the result has to be divided by 2 to get the correct per-image variance.

This process is repeated for different illumination levels by reducing the current through the LED to get multiple data points for the signal-variance plot. A linear curve can be fitted to those points, the slope of which equals g, the system gain in electrons per ADU.

In the second part, a Shack-Hartmann setup using a microlens array, a laser source, and a CMOS detector was assembled. The laser beam was collimated to produce a plane wavefront that could be analyzed with the microlens array and thus set a reference for further measurements.
The microlens array was carefully aligned so that each sub-aperture had a sharp focal point - also called *centroid* - on the CMOS detector, indicated by a corresponding sharp dot on the screen. The positions of the undistorted centroids were determined and saved in a CSV file for later reference.
After saving the reference locations, an astigmatism lens was introduced into the optical path to distort the incoming wavefront (simulating a possible aspect of seeing). A second image was taken with the CMOS camera under otherwise identical initial conditions, and the position of the new, distorted centroids were extracted.

Comparing the positions of the new centroids to the reference returns the desired displacements from which the Shack-Hartmann gradients can be computed, which show the local wavefront tilt. Using the provided python script *„display_gradients.py"*, the gradients were computed and then visualized. This is one of the first steps towards the reconstruction of the initial wavefront, which will not be part of this report.

3. Results

3.1 System Gain

The signal-variance plot was used to determine the system gain of the CMOS camera. For that, the variance was calculated by the provided python script *„gainSH_v1.py"*, which used the method elaborated upon in the previous chapter. It then plotted the evaluated data points as seen in the following figure.

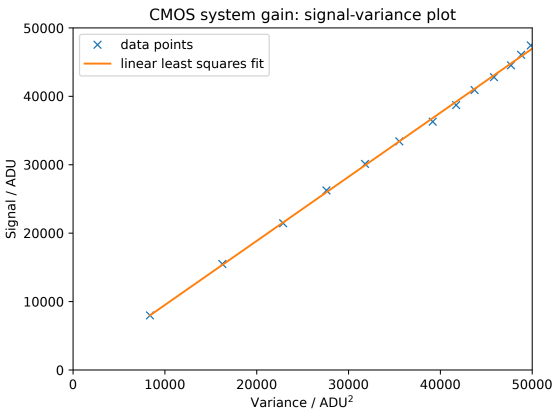

Figure 3.1: Signal - Variance Plot with slope g produced by *„gainSH_v1.py"*

The slope, which equals the system gain, was determined it to be:

$$g = 0,94 \, \frac{e^-}{\text{ADU}}.$$

This matches the example value given in the script [2]:

$$g_{script} = 0,944 \, \frac{e^-}{\text{ADU}}.$$

Knowing the gain allows for physical interpretation of the detector output.

3.2 Shack-Hartmann Sensor

To determine the gradients of a wavefront, the centroids of an undistorted wavefront were first obtained using *„find_centroids_and_save_results.py"*. Then, the setup was altered and an astigmatism lens was introduced. The centroids changed accordingly, which was also recorded with the same python script.

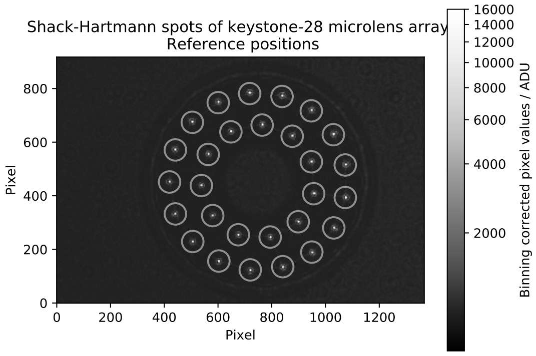

Figure 3.2: Reference Centroids

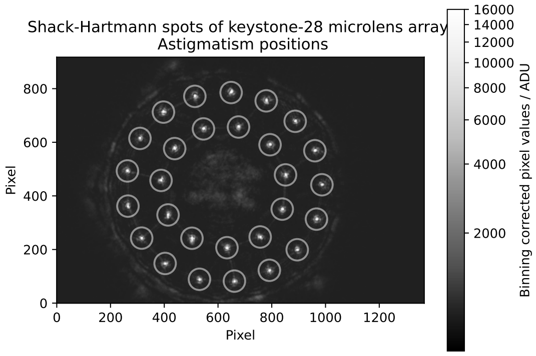

Figure 3.3: Astigmatism Centroids

The positions of these centroids were then used to calculate the wavefront gradients, which were plotted using *display_gradients.py*:

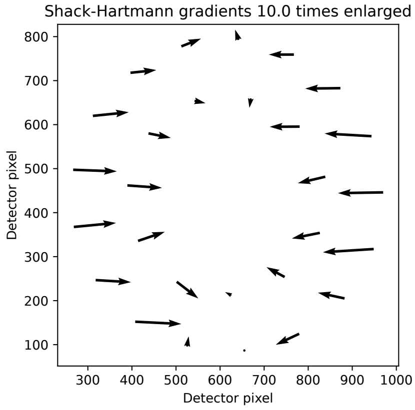

Figure 3.4: Astigmatism Gradients

While this plot has no spherical symmetry, two axes of mirror symmetry are visible, one vertical axis and another horizontal one. This suggests cylindrical symmetry, with the vertical axis aligning with the undistorted case and the horizontal axis appearing „stretched".

4. Discussion

4.1 System Gain

The system gain computed by the linear fit of the signal-variance plot returned a result of $g = 0,94\ \mathrm{e}^-/\mathrm{ADU}$. This result strongly coincides with the value provided by the script for this experiment, $g_{script} = 0,944\ \mathrm{e}^-/\mathrm{ADU}$. We thus have no reason to doubt the accuracy of this measurement.

4.2 Shack-Hartmann Sensor

All 28 centroids were able to be found, both for the reference as well as the image with the built-in refractive error.

While rotational symmetry is no longer observable, the gradients still appear to be mirror symmetric around a vertical and a horizontal axis. Since astigmatism is a refractive error caused by rotational asymmetry, the increase of gradient magnitude further away from the vertical symmetry axis combined with a decrease of it further away from the horizontal one was to be expected. Considering the symmetry of the gradients, it also becomes apparent as to why a cylindrical lens would be used to correct this.

The light scatter of the direction of the gradients can be explained by experimental inaccuracies such as alignment of optical components by hand, the lens not being a perfectly crafted astigmatism lens or simply being a bit dirty.

Bibliography

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[2] Markus Feldt: *Script for experiment F36 – »Wavefront analysis with a Shack-Hartmann sensor«*. Universität Heidelberg, 2021 https://www2.mpia-hd.mpg.de/AO/INSTRUMENTS/FPRAKT/F36.pdf. – (Last accessed on 28.05.2025, 17:29)

[3] Paul A. Tipler, Gene Mosca: *Physik für Studierende der Naturwissenschaften und Technik*. Springer, 2024 http://dx.doi.org/10.1007/978-3-662-67936-4