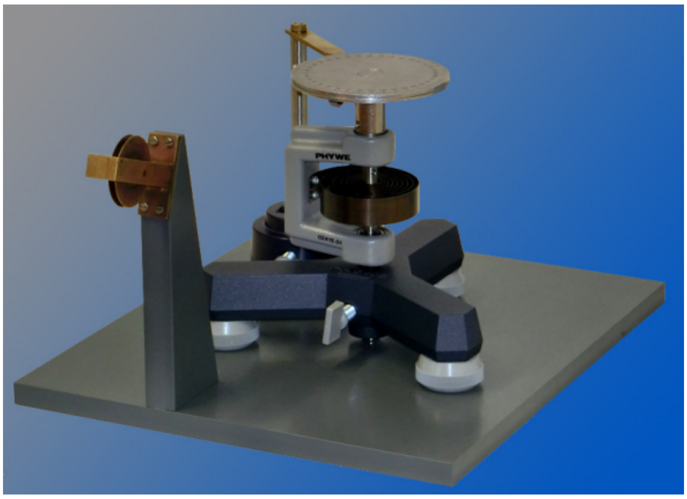
Experiment 12 - Moment of inertia

PAP 1, [2] [1]

02.09.2024



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Group: 1

Course: Afternoon

Contents

1. Introduction

1.1 Motivation

In this experiment we want to experimentally determine the moment of inertia of various objects. The moment of inertia is looked at for both regular and irregularly shaped bodies. The measurement is carried out by analyzing the period of oscillation of a rotating pendulum, whereby the physical variables such as torque and period of oscillation are noted and their inaccuracy is calculated. This experiment is intended to deepen the understanding of rotational dynamics, particularly with regard to the moment of inertia.

1.2 Measurements

The measurements are carried out using a pendulum around an axis. First of all, the pendulum's directing moment is determined by applying a tangential force to an aluminium-disk mounted on the axis of rotation. Then as a second method the period of oscillation of the pendulum is measured with different masses applied. By evaluating the period of oscillation and using known formulas, the moment of inertia of the bodies can be calculated.

1.3 Basics

1.3.1 Moment of inertia

The moment of inertia J has a central role in rotational dynamics and describes the resistance of a body to a change in its rotation. It depends on both the mass of the body and its distribution relative to the axis of rotation. Mathematically, the moment of inertia for a body is given by the formula:

$$J = \int r^2 dm \tag{1.1}$$

1.3.2 Torque

The torque M acting on the body is linked to the angular acceleration $\ddot{\varphi}$ by the torque law:

$$M = J \cdot \ddot{\varphi} \tag{1.2}$$

1.3.3 Directional Moment

In the experiment, the directing moment D of the torsion pendulum is determined experimentally. The formular here is:

$$M = -D \cdot \varphi \tag{1.3}$$

where φ is the angle of deflection. The directional torque D can be determined using the measured torques and deflection angles.

1.3.4 Period of oscillation

The period of oscillation T of such pendulum depends on the moment of inertia J and the directing moment D. The following applies to a pendulum:

$$T = 2\pi\sqrt{\frac{J}{D}} \tag{1.4}$$

By measuring the period of oscillation, one can draw conclusions about the moment of inertia of a body.

1.3.5 Steiner's theorem

Steiner's theorem is used to calculate the moment of inertia of a body about an axis that does not run through the center of gravity:

$$J = J_s + m \cdot d^2 \tag{1.5}$$

where J_s is the moment of inertia of the body through the center of gravity, m is the mass of the body and d is the distance between the center of gravity axis and the axis under investigation.

2. Execution of the experiment

2.1 Experimental Setup

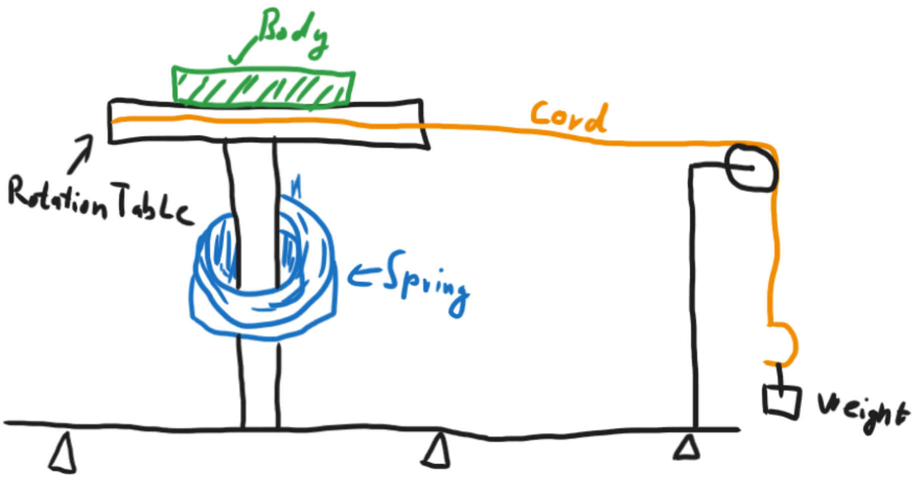

Figure 2.1: Sketch of the experiment

2.2 Measurements

Measurement protocol

2.9.2024
13:30 Uhr – 16:30 Uhr

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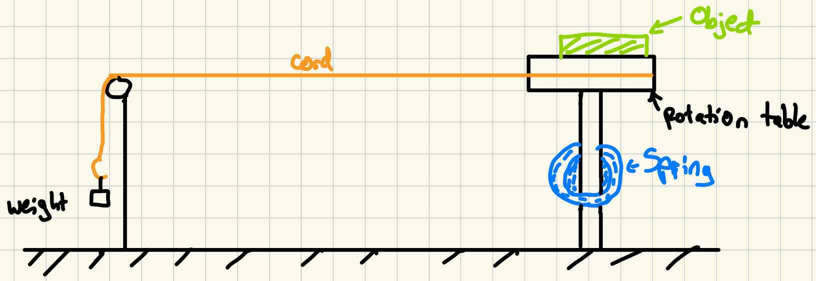
Experiment 12 – Moments of inertia

1. measurement setup:
   - Rotary pendulum with vertical axis
   - Rotaryfork and -table
   - Scale
   - Stopwatch and calipers
   - Balancing blade

2. Sketch:



3. Measuring the deflection angle created by the gravitational force and determining the directing torque D of the rotary pendulum

Table 1: Determination of the direction torque D

diameter aluminium-disc: 100mm ± 0,1mm

angle inaccuracy: ±2°
↳ the rotation table was scaled in 2° steps

Nr.	mass [g]	deflection angle [deg]
1	40	48
2	80	98
3	120	147
4	160	196
5	200	244
6	240	299

mass inaccuracy: ±1g
↳ the scale was scaled in 1g steps

4. Determination of the direction torque D from the oscillation period of a plate with known moment of inertia I_s

Table 2: Determination of the direction torque D

mass [g]	Nr.	measurement (20 oscillations) [s]	oscillation period T [s]	$\bar{T}$ [s]	$\sigma_{\bar{T}}$ [s]
without mass	1	24,32	1,22	1,21	0,01
	2	24,06	1,20		
	3	24,23	1,21		
with brass table	1	31,12	1,56	1,55	0,01
	2	31,09	1,55		
	3	31,01	1,55		

diameter of the brass table: 100mm ± 0,1mm → resolution of scale

mass of the brass table: 489g ± 1g

inaccuracy caused by reaction time: 0,2s

5. Measurements regarding moments of inertia

Table 3: measurements of the moments of inertia

(I) regarding the axis through the center of gravity

measurement (20 periods) [s]	period T [s]
45,68	2,28

(II) regarding the axes parallel to the center of gravity axis

Nr.	distance to c.o.g [mm]	measurement (20 periods) [s]	period T [s]
1	10	46,46	2,32
2	20	47,42	2,37
3	30	50,28	2,51
4	40	52,62	2,63
5	50	56,87	2,84

Inaccuracy of the distance measurement: 1mm

Mass of the plate: 683g ± 1g

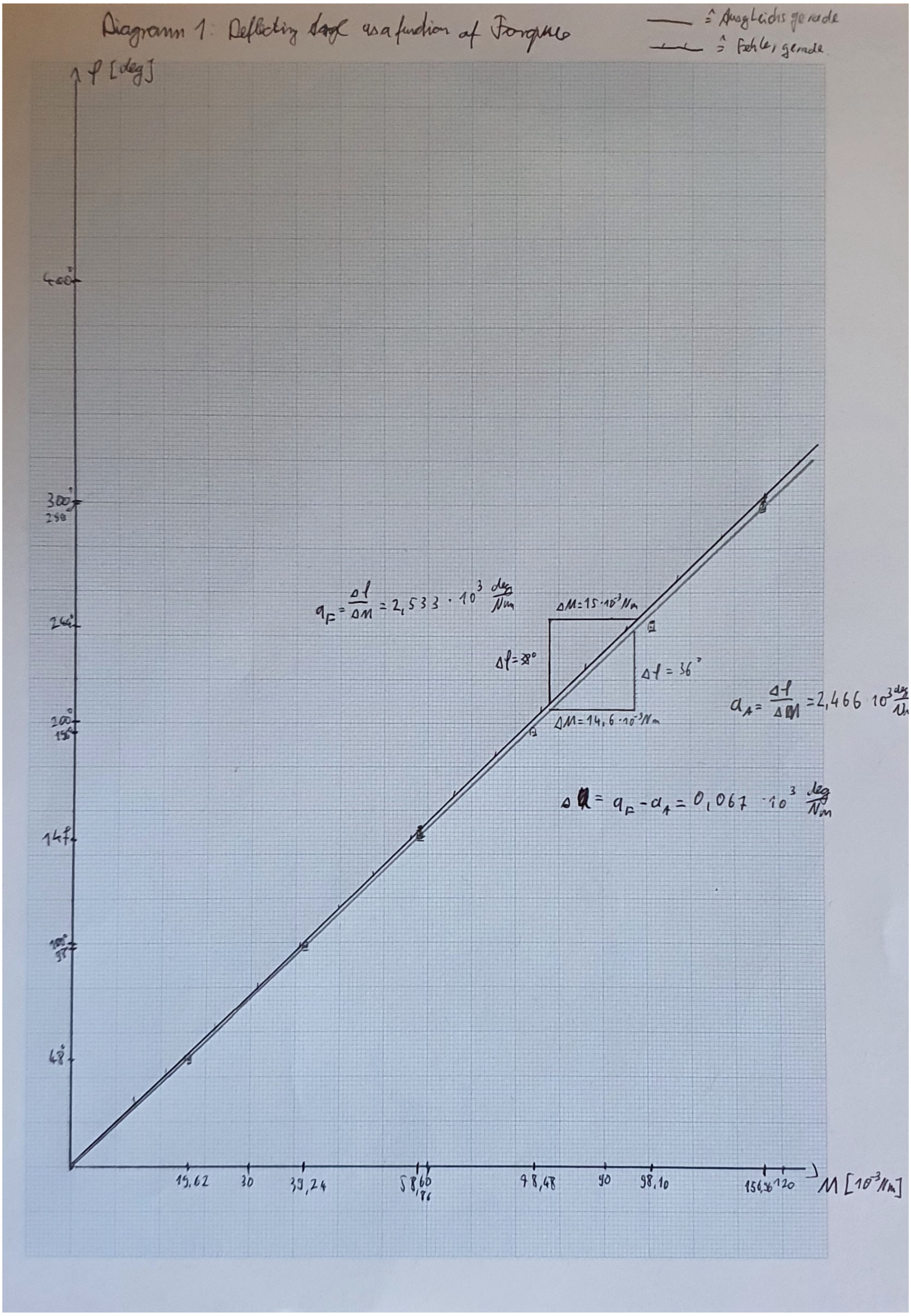


Diagramm 1: Deflecting angle as a function of Torque
≙ Ausgleichsgerade
≙ Fehlergerade
φ [deg]
400°
300°
290°
244°
200°
196°
147°
100°
98°
48°
19,62
30
39,24
58,86
60
78,48
90
98,10
156,96
120
M [10^-3 Nm]
a_F = Δφ/ΔM = 2,533 · 10^3 deg/Nm
ΔM = 15 · 10^-3 Nm
Δφ = 38°
Δφ = 36°
ΔM = 14,6 · 10^-3 Nm
a_A = Δφ/ΔM = 2,466 10^3 deg/Nm
Δa = a_F - a_A = 0,067 · 10^3 deg/Nm

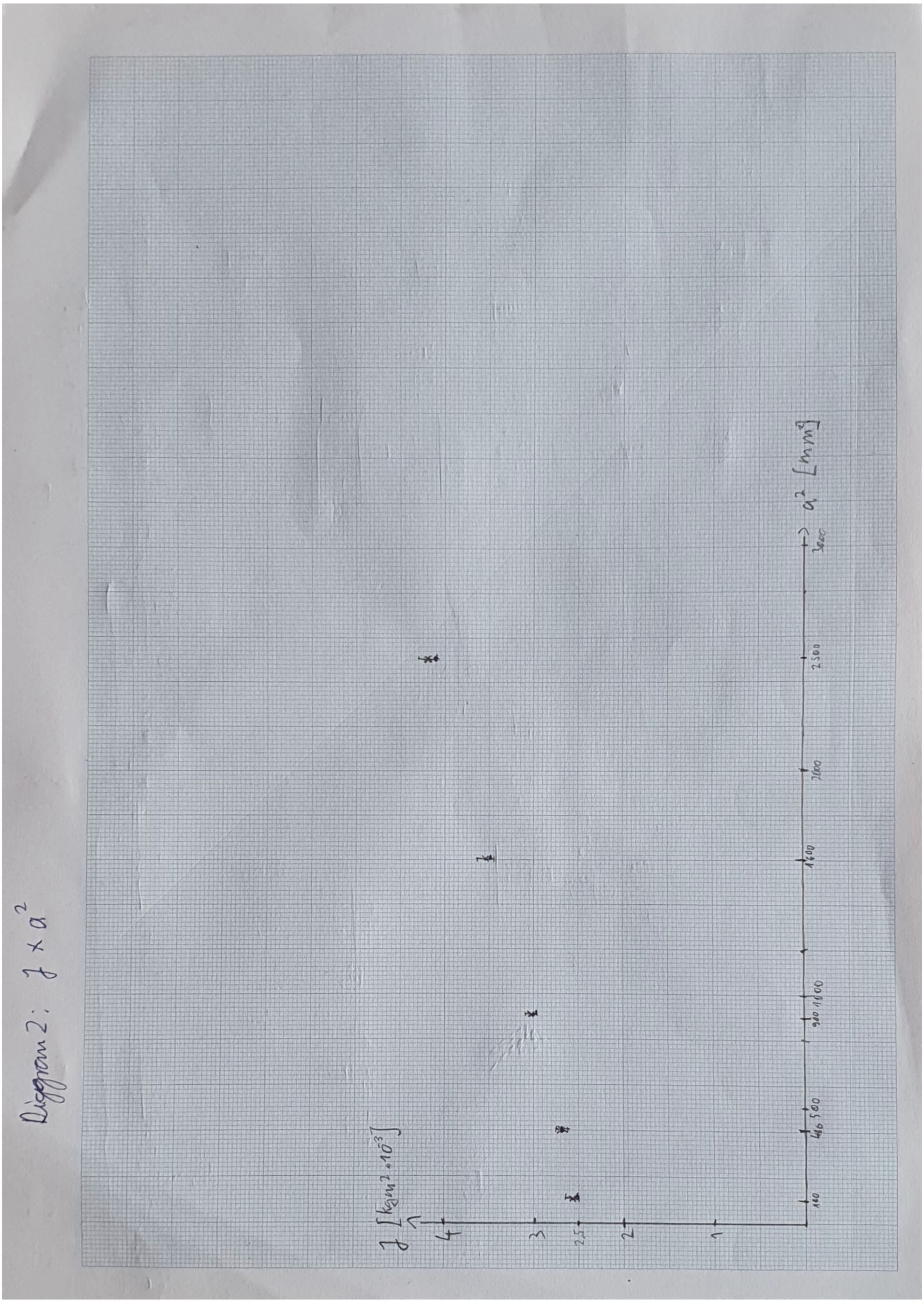


Diagramm 2: J x a²
J [kgm²·10⁻³]
a² [mm²]
4
3
2,5
2
1
100
400 500
900 1000
1600
2000
2500

3. Evaluation

3.1 Determination of directional Torque D

To determine the directional Moment, we can use a diagram, where we plot the measured angle as a function of the Torque .

We get the following Table:

Nr.	deflection angle φ in [deg]	Torque M in [Nm 10^{-3}]
1	48 ± 2	$19,62 \pm 0,5$
2	98 ± 2	$39,24 \pm 0,5$
3	147 ± 2	$58,86 \pm 0.5$
4	196 ± 2	$78,48 \pm 0,5$
5	244 ± 2	$98,10 \pm 0,5$
6	299 ± 2	$117,72 \pm 0,5$

Table 3.1: measured deflection angle and torque

The values for the torque where optained by using the following equation:

$$M = m \cdot g \cdot R \tag{3.1}$$

whereby R is $\frac{1}{2}d$, and d is the Diameter of the Aluminium-disk.

Since the values for m and r are measured and have some inaccuracy to them, as displayed in the Protokol, we need to consider the inaccuracy of the Torque ΔM as well.

$$\Delta M = \sqrt{(mg \cdot \Delta R)^2 + (gR \cdot \Delta m)^2} \tag{3.2}$$

3.1.1 Evaluation of Diagram 1

In the Diagramm 1 we determined graphicaly the gradient of the slope and its Error.

Using Formula 1.3:

$$D = \frac{M}{\varphi} = \frac{1}{a}, \quad \text{and } \Delta D = \frac{\Delta a}{a^2} \tag{3.3}$$

and the value for $a = 2,466 \cdot 10^3 \frac{deg}{Nm}$ as well as $\Delta a = 0,067 \cdot 10^3 \frac{deg}{Nm}$

We can calculate D as

$$\underline{D_1 = (4,06 \pm 0,11) \cdot 10^{-4} \frac{Nm}{deg}} = (2,33 \pm 0,06) \cdot 10^{-2} \frac{Nm}{rad}$$

3.2 Theoretical determination of the moment of inertia D

The moment of intertia for a disk can be calculated with:

$$J_s = \frac{1}{2} m r^2 \tag{3.4}$$

where m and r are the mass of the disk and the radius.

If the moment of inertia of the Table is J_T, then

$$T_1 = 2\pi\sqrt{\frac{J_T}{D}} \tag{3.5}$$

and

$$T_2 = 2\pi\sqrt{\frac{J_T + J_s}{D}} \tag{3.6}$$

Using the Formulas above, we can calculate the Formular for the directing moment as:

$$D = \frac{2\pi^2 mr^2}{T_2^2 - T_1^2} \tag{3.7}$$

Since m, r, T_2 and T_1 have errors themselves we need to consider the total error of D with the gaussian error formula. Using this, the error can be calculated with:

$$\Delta D = \sqrt{\left(\frac{-2\pi^2 r^2}{T_2^2 - T_1^2} \cdot \Delta m\right)^2 + \left(\frac{-4\pi^2 mr}{T_2^2 - T_1^2} \cdot \Delta r\right)^2 + \left(\frac{4T_2\pi^2 mr^2}{(T_2^2 - T_1^2)^2} \cdot \Delta T_2\right)^2 + \left(\frac{4T_1\pi^2 mr^2}{(T_2^2 - T_1^2)^2} \cdot \Delta T_1\right)^2} \tag{3.8}$$

Using this we can calculate D to be:

$$\underline{D_2 = (2,57 \pm 0,11) \cdot 10^{-2} \frac{Nm}{rad}}$$

3.3 Calculating the moment of inertia of the unregular plate

Using formula 3.6 and 1.4 we can rearange the formular to J_{plate}, where J_{plate} is the moment of inertia we are interested in.

$$J_{plate} = \frac{D}{4\pi^2}(T_2^2 - T_1^2) \tag{3.9}$$

with T_2 being the oscillation time of the table with the irregular body attached and T_1 the oscialltion time of just the Table.

Again, the measurement have errors, so we need to calculate the total error with:

$$\Delta J_{plate} = \sqrt{\left(\frac{T_2^2 - T_1^2}{4\pi^2} \cdot \Delta D\right)^2 + \left(\frac{2T_2 D}{4\pi^2} \cdot \Delta T_2\right)^2 + \left(\frac{2T_1 D}{4\pi^2} \cdot \Delta T_1\right)^2} \tag{3.10}$$

Using those Formula we get:

$$J_{plate} = (2,43 \pm 0,11) \cdot 10^{-3} kg \cdot m^2$$

3.4 Steiner's theorem

With the help of Steiners theorem, described in 1.5 and the resulting error formular:

$$\Delta J = \sqrt{(\Delta J_{plate})^2 + (d^2 \cdot \Delta m)^2 + (2md \cdot \Delta d)^2} \tag{3.11}$$

we obtain the following table.

Distance to CoG in [mm]	moments of inertia (measured) in $[kg \cdot m^2 \cdot 10^{-3}]$	moment of inertia (steiner) $[kg \cdot m^2 \cdot 10^{-3}]$
10 ± 1	$2,55 \pm 0,11$	$2.50 \pm 0,11$
20 ± 1	$2,70 \pm 0,12$	$2.70 \pm 0,11$
30 ± 1	$3,15 \pm 0,14$	$3.04 \pm 0,12$
40 ± 1	$3,55 \pm 0,15$	$3.52 \pm 0,12$
50 ± 1	$4,30 \pm 0,18$	$4.14 \pm 0,13$

Table 3.4: Steiners Theorem vs. measured moments of inertia

4. Conclusion

Using the deflection angle and the calculated torque, we were able to determine the directional torque graphically. The value we measured is

$$D = (2.33 \pm 0.06) \cdot 10^{-2} \frac{Nm}{rad}$$

Furthermore, we were also able to determine the directional moment by means of the period duration of oscillations, which were measured once with a brass disk and once without a brass disk. With these measurements we obtained a directional moment of

$$D = (2.57 \pm 0.11) \cdot 10^{-2} \frac{Nm}{rad}$$

If we compare the values of the two measurement methods, we obtain a deviation of 1.91σ, which is within the expected deviation range.

Using the directional moment of the torsion pendulum, we were now also able to determine the moment of inertia of an irregular object and obtained a value of

$$J_{plate} = (2.43 \pm 0.11) \cdot 10^{-3} kg \cdot m^2$$

The moment of inertia of the irregular body was then measured with respect to 5 other axes different from the center of gravity axis and then calculated using Steiner's theorem. If you compare the values with the largest difference, this deviation is only 0.73σ. We were therefore able to confirm Steiner's theorem with our experiment.

Bibliography

[1] Captain Joni: *pap1-tex-vorlage*. https://github.com/captain-joni/pap1-tex-vorlage. – [Online; Stand 28.08.2024]

[2] Dr. J.Wagner: *Physikalisches Praktikum 1 f¨ur Studierende der Physik B.Sc.* https://www.physi.uni-heidelberg.de/Einrichtungen/AP/info/Corona/PAP1.pdf. – [Online; Stand 01/2014]